"Contradictions"

by

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"It can't be! This is the fifth time!"

The young mathematician was furious. His face would have been purple, if only the video coding algorithm hadn't considered such a color impossible and had on its own decided to recolor the entire image, giving him an even less probable shade of orange. On the other end of the 6G connection, the Section Head's face was impassive, sporting a perfect ebony tone.

"Of course it can't be!", the Section Head replied icily. "That's the fifth Computational Logic Unit you've managed to destroy. I've told you over and over again: these Self-Adaptive Units are experimental, and we don't fully know how they'll behave in non-standard conditions. You were forbidden to continue your research, yet you persisted in going on with it, even falsifying your credentials so you could still log in to the Grid. We have no other choice but to terminate your connection interface."

"But…! Are you telling me… This is outrageous! How will I be able to complete my research? I would have been able to prove the Theorem of Inconsistency by now, if the Grid's administrators were capable of making these Self-Adaptive Units function properly!"

"No ifs, ands or buts! If you really think you're the Gödel of the twenty-first century, start doing your computations by hand. Ha! Good luck with your work, amanuensis ..."

The Section Head's hologram vanished. What's worse, the Grid connection desktop also disappeared. They had really done it! Lucky for me I saved the text of the initial part of my work, the young mathematician thought. Did that woman think she was being ironic, challenging me to work by hand? Well I'll show her what a Real Mathematician is capable of!

The Theorem of Inconsistency would be the culmination and crowning glory of the lengthy process leading to the destruction of mathematical certainties. It had all started a century ago with the publication of the famous theorems by Kurt Gödel which demonstrated that any language powerful enough to be able to perform the usual arithmetic operations had a problem: a theorem can be defined as true within that language, but it cannot be proven without "leaving the system", that is, adding a new ad hoc postulate. But this addition would generate new unprovable theorems, and so on ad infinitum: somewhat like the paradox of Achilles and the tortoise, one would be forced to complete a series of required steps without ever approaching the end. Mathematicians of the twentieth century were initially disconcerted, fearing that their whole castle of formulations and proofs might unhappily collapse: but human beings are experts at adapting to anything, and in a few years they had gone from asking "true or false?" to a less absolutist "true, false or undecidable?" whenever they had to analyze a mathematical statement.

Gödel's theorem, however, was more subtle than what the general public had read in popular magazines. In fact it affirmed the existence of unprovable theorems based on a very specific assumption: that mathematics was consistent, that is, that it was not possible to prove one thing and its opposite at the same time. No mathematician in full possession of his mental faculties – though talking about mathematicians, such a statement is rather dicey – believed that mathematics could be anything but consistent. After all, how could two plus two ever equal five? Yet the young mathematician had a hunch. Very long chains of mathematical inferences could take different paths through a problem's space and find themselves at the same point but with opposite truth values, a little like a pair of ants crawling in opposite directions on a Möbius strip: they would eventually meet, though one would have become the mirror image of the other. Such a discovery would revolutionize mathematics as we know it, and would lead straight to the Abel Prize, the Nobel Prize for mathematics, and who knows what else...

Without the use of the Grid, the undertaking seemed beyond anyone's reach, but luckily for him, the mathematician had saved two promising inferential chains: according to his estimates, he would just have to add about fifty more steps to the 1,250 sequences he already had in order to arrive at the point of contradiction. A job that the Grid would have computed in a day, but that even by hand would take a couple of months at most. The mathematician set to work feverishly, rechecking each step three times. He spent a sleepless final night: the first chain was now complete, and all he had to do now was write the last step of the second chain, to obtain the longed for contradiction. The signs and notations formed an orderly line and contradiction was just up ahead, almost personified: he just had to transcribe one hundred more digits, then fifty, twenty, ten, five, three, two, one. The young man was trembling as he checked his work one last time: the steps were all correct, and foolproof. There was no doubt about it: mathematics is
contradic[redacted]. [redacted]

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The entity observed the toy universe that was rapidly disintegrating. The waves of its vibrations no longer had any regularity: irrational frequencies continued to reverberate and the entity was unable to dampen them.

"Of course it can't be!", the other entity replied icily, as the permeation swiftly diminished. "That's the fifth universe that you've managed to destroy. I explained to you that you wouldn't be able to arrive at anything stable with those arithmetical laws. But no, you had to persevere, and you even tried to cheat by entering a self-locking routine as soon as a computer found a basic contradiction, to circumvent the impossibility of constructing a contradictory universe ... Yet it's a fundamental concept: as soon as a contradiction appears at any point of the universe, it propagates instantly and there isn't time to stop the destruction. I'm sorry: you've failed once again."